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Video 8

Ex- Show that $5^{38} \equiv 4 \pmod{11}$

Proof:- Since 11 is a prime and $(5,11)=1$ therefore we have

$5^{11} \equiv 5 \pmod{11}$ (Fermat's thm.)

$\therefore 5^{33} \equiv 5^3 \pmod{11}$

Now $5^{38} \equiv 5^{33} \times 5^5$

$\equiv 5^3 \times 5^5 \pmod{11}$

$\equiv 5^8 \pmod{11}$

$\equiv (5^2)^4 \pmod{11}$

$\equiv 3^4 \pmod{11}$

$\equiv 81 \pmod{11}$

$\equiv 4 \pmod{11}$

Ex:- Find the remainder when 41^{75} is divided by 3.

Proof:- we have $41 \equiv 2 \pmod 3$

$\therefore 41^{75} \equiv 2^{75} \pmod 3$

Now $(2,3)=1$ and 3 is a prime. Thus by Fermat's thm. we have

$2^2 \equiv 1 \pmod 3$

$\Rightarrow 2^{74} \equiv 1 \pmod 3$

Now $41^{75} \equiv 2^{75} \pmod 3$

$\equiv 2^{74} \times 2 \pmod 3$

$\equiv 1 \times 2 \pmod 3$

$\equiv 2 \pmod 3$

This shows that the remainder is 2 when 41^{75} is divided by 3.

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Ex:- Find the remainder when 5^{11} is divided by 7.

Proof:- we have $(5,7)=1$ and 7 is a prime.

Then, $5^6 \equiv 1 \pmod 7$

Also $5^4 \equiv 2 \pmod 7$

Now $5^{11} \equiv 5^6 \times 5^4 \times 5 \pmod 7$

$\equiv 1 \times 2 \times 5 \pmod 7$

$\equiv 10 \pmod 7$

$\equiv 3 \pmod 7$

The required remainder is 3.

Absolute Pseudo Primes :- (Carmichael Numbers)

A composite number n is called an absolute pseudo prime if $a^n \equiv a \pmod n$ for all integers a.

Ex:- Show that 1729 is an absolute pseudo prime

we have $1729 = 7 \cdot 13 \cdot 19$. If $(a, 1729) = 1$ then $(a,7)=1$, $(a,13)=1$ and $(a,19)=1$

Then by Fermat's the. we have

$a^6 \equiv 1 \pmod 7$, $a^{12} \equiv 1 \pmod{13}$, $a^{18} \equiv 1 \pmod{19}$

$a^{1728} = (a^6)^{288} \equiv 1 \pmod 7$

$a^{1728} = (a^{12})^{144} \equiv 1 \pmod{13}$

$a^{1728} = (a^{18})^{96} \equiv 1 \pmod{19}$

$\Rightarrow a^{1728} \equiv 1 \pmod{7 \cdot 13 \cdot 19}$

$\equiv 1 \pmod{1729}$

$\Rightarrow a^{1729} \equiv a \pmod{1729}$

$\Rightarrow$ 1729 is an absolute pseudo prime.